A NOTE ON CALCULATION OF TURBULENT HEAT TRANSFER ON A SEMI-PERMEABLE SURFACE WITH INJECTION OF AN OUTSIDE GAS

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An examination is made of a method of calculating the heat transfer on a semi-permeable surface washed by a subsonic stream with injection of outside gas, based on the solution of the energy equation using the asymptotic theory of the turbulent boundary layer. A comparison of the proposed method with experimental data is made.

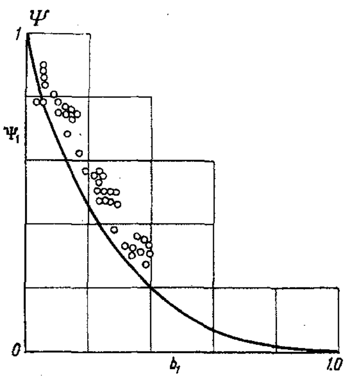

Fig. 1

The energy equation for a two-dimensional thermal boundary layer in the presence of a transverse stream of material (neglecting thermodiffusion and barodiffusion) may be written in the form

$$\frac{dR_1^{**}}{dX} + R_1^{**}\frac{d\Delta i}{\Delta i dX} = R_+ S_0(\Psi_1 + b_1),$$

$$R_1^{**} = \frac{\delta_1^{**} w_0}{\nu_0}, \quad R_+ = \frac{w_0 l}{\nu_0}, \quad X = \frac{x}{l}, \quad \Psi_1 = \frac{S_1}{S_{01}},$$

$$\delta_1^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{i - i_w}{i_0 - i_w}\right) dy, \quad \Delta i = i_0 - i_w \quad i_0 = T_0 c_{p0}, \quad i_w = T_w c_{pw}. \qquad (1)$$

Here δ_1^{**} is the energy thickness, i_0, i_w is the enthalpy of the stream outside the boundary layer and on the porous wall; T_0, c_{p0}, T_w, c_{pw} are the temperature and specific heats outside the boundary layer and at the wall; ρ, ρ_0, w, w_0 are the density and velocity in the boundary layer and at its edge; S_1 is the Stanton number; ν_0 is the kinematic viscosity at the edge of the boundary layer at temperature T_0. For a standard Stanton number value in the region of the Reynolds number R_1^{**} of practical interest, we may take the well-known formula

$$S_{01} = 0.0126\,(R_1^{**})^{-0.25} P^{-0.75}. \qquad (2)$$

By S_1 we mean the ratio

$$S_1 = \frac{q}{\gamma_0 w_0 (i_0 - i_w)} \qquad \left(q = -\frac{\lambda}{c_p}\frac{di}{dy}\right). \qquad (3)$$

Here q is the total thermal energy flux to the body surface; an expression for q when the Lewis number Le = 1 is given in parentheses. The injection parameter is

$$b_1 = \frac{j_w}{S_{01}} \qquad \left(j_w = \frac{\rho_w w_w}{\rho_0 w_0}\right). \qquad (4)$$

Here j_w is the relative flow rate of injected gas.

For values of Prandtl number P = 1 and of Lewis number Le = 1, in the case of flow over a plate without pressure gradient and with similarity of boundary conditions, the equations of momentum and energy (1) become identical, and therefore, when $R_1^{**} = R^{**}$,

$$S_{01} = \frac{C_{f0}}{2}, \quad b_1 = b = j_w\frac{2}{C_{f0}}, \quad R^{**} = \frac{w_0\delta^{**}}{\nu_0}. \qquad (5)$$

Here δ^{**} is the momentum thickness. The expressions in (5), in turn, lead to $\Psi_1 = \Psi$, where the quantity $\Psi = C_f/C_{f0}$ was defined in [1].

Taking account of Eqs. (5), we obtain

$$\Psi_1 = \Psi = \left(\frac{2}{\sqrt{\psi_1}+1}\right)^2\left(1 - \frac{b_1}{b_{1*}}\right)^2. \qquad (6)$$

Here b_{1*} is the critical injection parameter. To determine b_{1*}, we may use the formula obtained in [1], or the approximation for it proposed by Spalding:

$$b_{1*} = \frac{4}{1/3 + 2/3\psi_1}. \qquad (7)$$

According to [1],

$$\psi_1 = \psi\left[1 + \frac{b_{11}}{1+b_{11}}(R-1)\right] = \frac{\rho_0}{\rho_w}$$

$$\left(\psi = \frac{T_w}{T_0}, \ R = \frac{R_1}{R_0}, \ b_{11} = \frac{b_1}{\Psi}\right). \qquad (8)$$

Here R_0, R_1 are the gas constants of the main and the injected gases.

From the energy balance at the surface of the porous plate we evidently obtain the equality

$$\Psi_1 = b_1 k_1 \qquad \left(k_1 = \frac{i_w - i_1}{i_w - i_0}\right). \qquad (9)$$

Here i_1 is the enthalpy of the injected gas, and T_1 and c_{p1} are its temperature and specific heat. Thus, the system of equations (1), (2), (4), (6)–(8) becomes closed and may be solved with respect to all quantities of interest.

Allowing for Eq. (9), we may transform Eq. (8) to the form

$$\psi_1 = \psi\left[1 + \frac{1}{k_1+1}(R-1)\right]. \qquad (10)$$

The quantity k_1 depends on the value of i_w, which is a function of the injection parameter b_1 and may be unknown beforehand. The enthalpy at the wall is expressed by the formula

$$i_w = [c_{p1}\rho_{1w}^+ + c_{p0}(1 - \rho_{1w}^+)]\,T_w =$$

$$= T_w\,[\rho_{1w}^+(c_{p1} - c_{p0}) + c_{p0}]. \qquad (11)$$

Here ρ_{1w}^+ is the relative mass concentration of injected gas at the wall. Because of Eq. (5) and according to [1], we have

$$\rho_{1w}^+ = \frac{b_{11}}{1+b_{11}} \quad \text{or} \quad \rho_{1w}^+ = \frac{1}{1+k_1}. \qquad (12)$$

In the latter case and here Eqs. (8) and (9) have been taken into account. Substituting Eq. (12) into Eq. (11) in accordance with Eq. (9), we find

$$k_1 = k_2\frac{c_{p1}}{c_{p0}} \qquad \left(k_2 = \frac{T_w - T_1}{T_0 - T_w}\right). \qquad (13)$$

Usually determination of the mass flow of injected gas j_w is required to secure a given temperature on the porous surface T_w. The parameters w_0, T_0, c_{p0}, c_{p1}, T_1, R here are known. The problem is solved in the following order. From Eq. (13) k_2 and k_1 are found, then Ψ_1 from Eq. (10), and we find b_{1*} from Eq. (7). Thereafter solution of Eqs. (6) and (9) gives the value of b_1. Substituting Eq. (6) and b_1 into Eq. (1), we find R_1^{**}. Equations (2) and (5) give the desired value j_w. The effect of the finiteness of the numbers R_1^{**} on the function Ψ_1

is estimated in the first approximation via the quantity b_{1*}. To determine the dependence of b_{1*} on the number R_1^{**} we may use the method described in [1].

Figure 1 gives a comparison of the dependence of Ψ_1 on $b_1 = j_w/S_{01}$ with the experimental data of [2] on heat transfer on a porous surface with helium injection. It should be noted that the test data on the graph are referred to S_{01}, which was defined by formulas (1.2) for the corresponding values of R_1^{**}.

As is seen, the theory is in satisfactory agreement with the experimental data.

It is interesting to compare the method described above for calculation of heat transfer with the approximate method described in [1]. This approximate method is based on the energy equation

$$\frac{dR_2^{**}}{dX} + R_2^{**}\frac{d\Delta T}{\Delta T dX} = R_L S_0 (\Psi_2 + b_2),$$

$$R_2^{**} = \frac{\delta_2^{**} w_0}{\nu_0}, \qquad \delta_2^{**} = \int\limits_0^\infty \frac{\rho w c_p}{\rho_0 w_0 c_{p0}}\left(1 - \frac{T - T_w}{T_0 - T_w}\right) dy, \quad (14)$$

$$\Psi_2 = \frac{S}{S_0} = \left(\frac{2}{\sqrt{\psi} + 1}\right)^2 \left(1 - \frac{b_2}{b_{2*}}\right)^2, \quad (15)$$

$$\Psi_2 = b_2 k_2, \qquad b_2 = j_w \frac{c_{p1}}{c_{p0} S_0}, \quad (16)$$

$$S = \frac{q_1}{\gamma_0 w_0 c_{p0} \Delta T}, \qquad \Delta T = T_0 - T_w,$$

$$S_0 = 0.0126\,(R_2^{**})^{-0.25}\,P^{-0.75}. \quad (17)$$

Here q_1 is the heat flux (given to the wall), b_{2*} is the critical value of the injection parameter b_2 (at the point where the flow is pushed aside) which is found from the formulas of [1] as a function of the parameters ψ, R and R_2^{**}. When $c_{p1} = c_{p0}$ the systems (1)–(7) and (14)–(17) become identical. It is very interesting to compare the solutions by the two methods for substantially different heat capacities c_{p1} and c_{p0}. In essence this implies a check of the degree of conservatism of the laws of heat transfer in the energy Eqs. (1) and (14). As an example we chose the problem of determining the required amount of injected gas to secure a given value of wall temperature, constant along the surface T_w = const. The calculation was done for various k_2 and R (k_2 = 0.005–6.0, R = 0.25; 4, c_{p1}/c_{p0} = 0.25; 4, ψ = 0.303–0.9).

Figure 2 shows the variation of the ratio j_{w1}/j_w. Here for values c_{p1}/c_{p0} = 4, R = 4 (Curve 1) and c_{p1}/c_{p0} = 0.25, R = 0.25 (curve 2) the value of j_w implies a flow rate of injected gas found from solution of the system of Eqs. (14)–(17), and j_{w1}—from the system (1)–(13). As may be seen from the figure, the difference in the flow rate values is quite insignificant even for substantially different physical properties of the main and injected gases. On reaching critical injection (k_2 = 0) the difference reaches its maximum value. In this case, in accordance with Eqs. (1), (2), (4), (14), (16), and (17), at critical injection we have

$$\frac{j_*}{j_{*1}} = \left(\frac{c_{p0}}{c_{p1}}\right)^{0.2}. \quad (18)$$

Hence it is clear that for a ratio c_{p1}/c_{p0} = 5.25 (helium - air), the ratio of flow rates is small and is

$$j_* / j_{*1} = 0.73.$$

Thus, for injection of outside gases in the regimes $b_1 > b_{1*}$ the two methods give practically identical results. It was shown in [1] that

$${}^1\!/_2 C_f = S \quad \text{for } P = Le = 1. \quad (19)$$

It follows from Eqs. (19) and (5) that for the same conditions $S_1 = S$, or, taking into account Eqs. (1), (2), (14), (15), and (17),

$$\frac{\Psi_1 (\Psi_2 + b_2)^{0.2}}{(\Psi_1 + b_1)^{0.2}\Psi_2} = A \approx 1. \quad (20)$$

As is seen from Fig. 2, where curve 3 represents the relation $A = A(k_2)$ with c_{p1}/c_{p0} = 4, R = 4, Eq. (20) is actually observed over

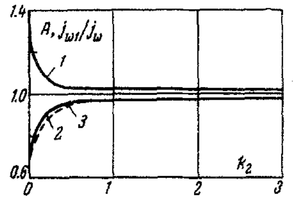

Fig. 2

practically the whole range of k_2. The largest discrepancies are observed at critical injection. In that case

$$A \to (b_2 / b_1)^{0.2} \to (c_{p1}/c_{p0})^{0.2}.$$

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